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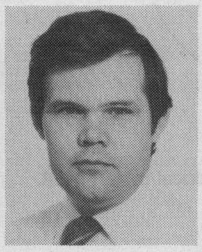

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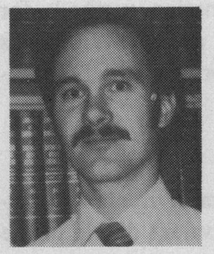

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Location of Sources of Evoked Scalp Potentials: Corrections for Skull and Scalp Thicknesses

JAMES P. ARY, STANLEY A. KLEIN, AND DEREK H. FENDER

Abstract—The problem of locating the position of the source of evoked potentials from measurements on the surface of the scalp has been examined. It is shown that the position of the source in a head modeled by a sphere surrounded by two concentric shells of differing conductivities representing the skull and the scalp can be inferred from source localization calculations made on a homogeneous model.

Sidman *et al.* [9] proposed an approximate calculation to achieve the same goal, but it is shown that while their approximation is very good for sources located near the center of the head, such as midbrain or brainstem structures, it is less satisfactory for sources at an eccentricity of 0.6-0.9, which is the location of most cortical sources. In fact, over this range, their correction may introduce as much error as it purports to remove.

It is shown that midrange estimates of skull thickness and scalp thickness may introduce localization errors of ±7 and ±3 percent of the outside radius of the scalp, respectively, but a poor estimate of skull conductivity introduces at most a 2 percent error.

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Introduction

Over the past decade several authors have reported attempts to calculate the locations of the cortical sources associated with the EEG. The sources have been modeled as current dipoles, and the head has been modeled either as a homogeneous sphere or as a concentric shell structure consisting of a homogeneous sphere of neural tissue surrounded by two concentric spherical shells of differing electrical conductivities representing the skull and scalp [1]-[5].

The calculation of the source parameters is usually an iterative numerical process. An initial set of source parameters is assumed, and the surface potential distribution over the model head is calculated. This distribution is then compared with the measured potential over the real head and the parameters of the source are changed to minimize the difference between the two distributions [6]-[8]. The more complex models lead to more realistic source parameters, but they carry a considerable penalty in terms of computer running time, sometimes by as much as a factor of 50. For this reason, a number of authors [9]; [10] have calculated the parameters of the source using the homogeneous model and then applied correction factors to obtain the parameters of the source in an inhomogeneous shell model.

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We shall show that these methods are based on an approximation that is not valid at the large values of eccentricity where most cortical sources are found. We will demonstrate the exact method and give adequate approximation methods for estimating the position of a source in an inhomogeneous model head based on computations performed on a homogeneous model.

Problem

We wish to consider the following. Given a dipole in an inhomogeneous model, can we find an equivalent dipole in the homogeneous model such that the field generated on the surface of the homogeneous model is the least-squared-error fit to the field generated on the surface of the inhomogeneous model.

Theory

Consider a dipole with radial and tangential components m_r and m_t embedded in a homogeneous sphere of radius R and conductivity σ. The coordinate system is shown in Fig. 1. The dipole is located at a distance z from the center of the sphere on the z axis and the dipole moment is in the positive xz plane; any other position or orientation of the dipole can be obtained by rotation. The potential $V(\alpha, \beta)$ at the surface of the sphere is given by

$$V(\alpha,\beta) = \frac{1}{4\pi\sigma} \sum_{n=1}^{\infty} \frac{2n+1}{n} b^{n-1} \left[n m_r P_n(\cos\alpha) + m_t P_n^1(\cos\alpha)\cos\beta \right] \quad (1)$$

where $b = z/R$ is the eccentricity of the dipole in the homogeneous model and where $P_n(\cos\alpha)$, $P_n^1(\cos\alpha)$ are Legendre and associated Legendre polynomials [11].

Next, consider a shell model consisting of a homogeneous sphere of neural tissue, radius r_1, surrounded by a concentric spherical shell of outside radius r_2 representing the skull, and another concentric spherical shell of outside radius R representing the tissue of the scalp. Let σ represent the conductivity of the neural tissue and of the scalp; these are assumed to have the same value. Let the conductivity of the skull be σ_s and let $\xi = \sigma_s/\sigma$. Then the potential $\overline{V}(\alpha, \beta)$ at the surface of the inhomogeneous model is given by

$$\overline{V}(\alpha,\beta) = \frac{1}{4\pi\sigma} \sum_{n=1}^{\infty} \frac{2n+1}{n} \overline{b}^{n-1} \left[\frac{\xi(2n+1)^2}{d_n(n+1)} \right] \cdot \left[n\overline{m}_r P_n(\cos\alpha) + \overline{m}_t P_n^1(\cos\alpha)\cos\beta \right] \quad (2)$$

where $\overline{b}$ = eccentricity of dipole in the inhomogeneous model, $\overline{m}_r$, $\overline{m}_t$ = radial tangential dipole moments in the inhomogeneous model, and

$$d_n = [(n+1)\xi + n] \left[\frac{n\xi}{n+1} + 1 \right] + (1-\xi)\,[(n+1)\xi + n] \cdot (f_1^{2n+1} - f_2^{2n+1}) - n(1-\xi)^2 (f_1/f_2)^{2n+1} \quad (2a)$$

where $f_1 = r_1/R$ and $f_2 = r_2/R$ [10], [12].

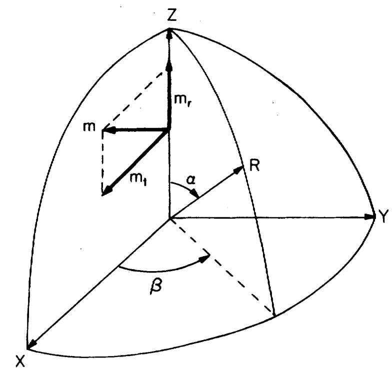

Fig. 1. Coordinate system for dipoles in spherical model head.

Radial Dipole

For a radially oriented dipole $\overline{m}_t = m_t = 0$, so (1) and (2) reduce to

$$V_r(\alpha,\beta) = \frac{1}{4\pi\sigma} \sum_{n=1}^{\infty} (2n+1) b^{n-1} m_r P_n(\cos\alpha)$$

and

$$\overline{V}_r(\alpha,\beta) = \frac{1}{4\pi\sigma} \sum_{n=1}^{\infty} (2n+1) \overline{b}^{n-1} F_n \overline{m}_r P_n(\cos\alpha) \quad (3)$$

where

$$F_n = \left[\frac{\xi(2n+1)^2}{d_n(n+1)} \right]. \quad (3a)$$

Our approach is to minimize ρ_r, the squared difference between surface potentials predicted by the two models and integrated over the sphere.

$$\rho_r = \int_0^{2\pi} \int_0^{\pi} [V_r(\alpha,\beta) - \overline{V}_r(\alpha,\beta)]^2 \sin\alpha \, d\alpha \, d\beta. \quad (4)$$

To evaluate (4) we substitute from (3) and collect the coefficients of the Legendre polynomials which gives

$$V_r(\alpha,\beta) - \overline{V}_r(\alpha,\beta) = \frac{1}{4\pi\sigma} \sum_{n=1}^{\infty} A_n P_n(\cos\alpha)$$

where

$$A_n = (2n+1)(b^{n-1} m_r - \overline{b}^{n-1} F_n \overline{m}_r)$$

and

$$\rho_r = \frac{1}{(4\pi\sigma)^2} \int_0^{2\pi} \int_0^{\pi} \left[\sum_{n=1}^{\infty} A_n P_n(\cos\alpha) \right]^2 \sin\alpha \, d\alpha \, d\beta.$$

The integration over β gives a factor of 2π, and the integration over α can be carried out using the orthogonal properties of the Legendre polynomials. Writing $\eta = \cos\alpha$ and substituting gives

$$\rho_r = \frac{1}{8\pi\sigma^2} \int_{-1}^{1} \left[\sum_{n=1}^{\infty} A_n P_n(\eta) \right]^2 d\eta.$$

Most of the terms of the integrand do not contribute to ρ_r because of the orthogonality condition

$$\int_{-1}^{1} P_j(\eta)P_k(\eta)\,d\eta = 0 \qquad \text{for } j \neq k$$

$$= \frac{2}{2n+1} \qquad \text{for } j = k.$$

Then $\rho_r = 1/(8\pi\sigma^2)\ \Sigma_{n=1}^{\infty}\ A_n^2\,[2/2n+1]$ and by substituting back for A_n we get

$$\rho_r = \frac{1}{4\pi\sigma^2}\sum_{n=1}^{\infty}(2n+1)\,[b^{n-1}m_r - F_n\bar{b}^{n-1}\overline{m}_r]^2. \tag{5}$$

To express this in terms of m_r, we write

$$\rho_r = \mu_{bb}m_r^2 - 2\mu_{bF}m_r + \mu_{FF} \tag{6}$$

where

$$\mu_{bb} = \frac{1}{4\pi\sigma^2}\sum_{n=1}^{\infty}(2n+1)b^{2n-2}$$

$$\mu_{bF} = \frac{1}{4\pi\sigma^2}\sum_{n=1}^{\infty}(2n+1)b^{n-1}\bar{b}^{n-1}F_n\overline{m}_r$$

$$\mu_{FF} = \frac{1}{4\pi\sigma^2}\sum_{n=1}^{\infty}(2n+1)\bar{b}^{2n-2}F_n^2\overline{m}_r^2. \tag{7}$$

The value of m_r that minimizes ρ_r can be found by setting $\partial\rho_r/\partial m_r = 0$, that is

$$\frac{\partial\rho_r}{\partial m_r} = 2m_r\mu_{bb} - 2\mu_{bF} = 0.$$

Let the value of m_r at the minimum be denoted by $\tilde{m}_r$; then

$$\tilde{m}_r = \mu_{bF}/\mu_{bb}. \tag{8}$$

The value of b that minimizes ρ_r can be found by setting $\partial\rho_r/\partial b = 0$. Inserting $\tilde{m}_r$ for m_r in (6) leads to

$$\rho_r = \mu_{FF} - [\mu_{bF}^2/\mu_{bb}]. \tag{9}$$

The calculation of $\partial\rho_r/\partial b$ is not easy, and the resultant equation does not have a simple analytic solution; however, it can be solved numerically. Some values of the minimum $\tilde{b}_r$ and the ratio $\overline{m}_r/\tilde{m}_r$ are given in Table I.

Tangential Dipole

Again the approach is to minimize the squared difference between surface potentials predicted by the two models and integrated over the sphere. In this case (1) and (2) reduce to

$$V_t(\alpha,\beta) = \frac{1}{4\pi\sigma}\sum_{n=1}^{\infty}\frac{2n+1}{n}b^{n-1}m_tP_n^1(\cos\alpha)\cos\beta$$

and

$$\overline{V}_t(\alpha,\beta) = \frac{1}{4\pi\sigma}\sum_{n=1}^{\infty}\frac{2n+1}{n}\bar{b}^{n-1}F_n\overline{m}_tP_n^1(\cos\alpha)\cos\beta. \tag{10}$$

Equation (4) becomes

$$\rho_t = \int_0^{2\pi}\int_0^{\pi}[V_t(\alpha,\beta) - \overline{V}_t(\alpha,\beta)]^2\sin\alpha\,d\alpha\,d\beta. \tag{11}$$

We use

$$[V_t(\alpha,\beta) - \overline{V}_t(\alpha,\beta)] = \frac{1}{4\pi\sigma}\sum_{n=1}^{\infty}A_nP_n^1(\cos\alpha)\cos\beta$$

where now

$$A_n = \frac{2n+1}{n}[b^{n-1}m_t - \bar{b}^{n-1}F_n\overline{m}_t]$$

so that (11) can be rewritten as

$$\rho_t = \frac{1}{(4\pi\sigma)^2}\int_0^{2\pi}\int_0^{\pi}\left[\sum_{n=1}^{\infty}A_nP_n^1(\cos\alpha)\cos\beta\right]^2 \cdot \sin\alpha\,d\alpha\,d\beta. \tag{12}$$

The integration over β gives a factor of π and the integration over α can be carried out using the orthogonality properties of the associated Legendre polynomial. Writing $\eta = \cos\alpha$ and substituting from (12) gives

$$\rho_t = \frac{1}{16\pi\sigma^2}\int_{-1}^{1}\left[\sum_{n=1}^{\infty}A_nP_n^1(\eta)\right]^2 d\eta.$$

For associated Legendre polynomials

$$\int_{-1}^{1}P_j^1(\eta)P_k^1(\eta)\,d\eta = 0 \qquad \text{for } j \neq k$$

$$= \frac{2n(n+1)}{2n+1} \qquad \text{for } j = k.$$

Therefore

$$\rho_t = \frac{1}{16\pi\sigma^2}\sum_{n=1}^{\infty}(A_n)^2\frac{2n(n+1)}{2n+1}$$

$$= \frac{1}{8\pi\sigma^2}\sum_{n=1}^{\infty}\frac{(2n+1)(n+1)}{n}[b^{n-1}m_t - \bar{b}^{n-1}F_n\overline{m}_t]^2$$

$$= \mu_{bb}^*m_t^2 - 2\mu_{bF}^*m_t + \mu_{FF}^* \tag{13}$$

where

$$\mu_{bb}^* = \frac{1}{8\pi\sigma^2}\sum_{n=1}^{\infty}\frac{(2n+1)(n+1)}{n}b^{2n-2}$$

$$\mu_{bF}^* = \frac{1}{8\pi\sigma^2}\sum_{n=1}^{\infty}\frac{(2n+1)(n+1)}{n}b^{n-1}\bar{b}^{n-1}F_n\overline{m}_t \tag{14}$$

and

$$\mu_{FF}^* = \frac{1}{8\pi\sigma^2}\sum_{n=1}^{\infty}\frac{(2n+1)(n+1)}{n}\bar{b}^{2n-2}F_n^2\overline{m}_t^2.$$

TABLE I

VALUES OF BEST-FIT DIPOLE PARAMETERS FOR A MODEL HEAD BASED ON THE PARAMETERS GIVEN BY RUSH AND DRISCOLL [13], NAMELY, r_1 = .87 r_2 = .92, ξ = .0125. PARAMETERS DENOTED BY A BAR, E.G., $\bar{b}$ REFER TO A DIPOLE IN A THREE-SHELL MODEL AND PARAMETERS DENOTED BY A TILDE, E.G., $\tilde{b}$ REFER TO THE DIPOLE IN A HOMOGENEOUS MODEL THAT BEST FITS THE SURFACE FIELD DISTRIBUTION GENERATED BY THE SHELL-MODEL DIPOLE.

	Radial Dipole				Tangential Dipole			
$\bar{b}$	$\tilde{b}_r$	$b_r/\tilde{b}_r$	$b - \tilde{b}_r$	$\bar{m}_r/\tilde{m}_r$	$\tilde{b}_t$	$\bar{b}_t/\tilde{b}_t$	$\bar{b} - \tilde{b}_t$	$\bar{m}_t/\tilde{m}_t$
.01	.006146	1.627	.004	1.515	.006146	1.627	.004	1.515
.1	.062	1.624	.038	1.517	.062	1.624	.038	1.516
.2	.123	1.619	.077	1.520	.123	1.619	.077	1.519
.3	.186	1.610	.114	1.524	.186	1.610	.114	1.522
.4	.251	1.596	.149	1.529	.250	1.600	.150	1.525
.5	.317	1.576	.183	1.536	.316	1.582	.184	1.531
.6	.387	1.549	.213	1.550	.385	1.558	.215	1.539
.7	.463	1.510	.237	1.572	.460	1.522	.240	1.555
.72	.480	1.500	.240	1.580	.475	1.516	.245	1.560
.74	.497	1.490	.243	1.587	.492	1.504	.248	1.566
.76	.514	1.479	.246	1.597	.508	1.496	.252	1.572
.78	.532	1.466	.248	1.608	.526	1.483	.254	1.580
.80	.551	1.452	.249	1.621	.544	1.471	.256	1.589
.82	.570	1.437	.250	1.639	.562	1.459	.258	1.601
.84	.591	1.421	.249	1.661	.582	1.443	.258	1.615
.86	.613	1.401	.247	1.689	.603	1.426	.257	1.633

The minimizing value of m_t is given by

$$\tilde{m}_t = \mu^*_{bF}/\mu^*_{bb}. \tag{15}$$

Inserting $\tilde{m}_t$ from (15) into (13) gives

$$\rho_t = \mu^*_{FF} - [\mu^{*2}_{bF}/\mu^*_{bb}] \tag{16}$$

and once again, the minimizing values of b, which we call $\tilde{b}$, can be found numerically. Some characteristic values are given in Table I.

DISCUSSION

Both Schneider [9] and Sidman *et al.* [10] have applied the method originally developed by Arthur and Geselowitz [12], to calculate a correction factor for transforming homogeneous model source parameters to inhomogeneous shell model parameters. However, the Arthur and Geselowitz method explicitly ignored those terms beyond the quadrupole ($n > 2$) in the series expansions of $V(\alpha, \beta)$ and $\bar{V}(\alpha, \beta)$. This approximation is justified for the deep sources involved in cardiac modeling, but it introduces significant errors in modeling cortical sources where the eccentricity is large.

If we designate the Schneider and the Sidman *et al.* homogeneous dipole parameters by b^0, m_r^0, and m_t^0, then they state that

$$\bar{b}/b^0 = F_1/F_2 \tag{17}$$

and that

$$\bar{m}_r/m_r^0 = \bar{m}_t/m_t^0 = 1/F_1 \tag{18}$$

where F_n is given in (3a). Using the Rush and Driscoll [13] values of radii and conductivities for the human head, $\xi = 0.0125$, r_1 = 8.0 cm, r_2 = 8.5 cm, and R = 9.2 cm, Schneider and Sidman *et al.* find $\bar{b}/b^0 = 1.63$ and $\bar{m}_r/m_r^0 = \bar{m}_t/m_t^0 = 1.52$. Our corresponding values of $\bar{b}_r/\tilde{b}_r$, $\bar{m}_r/\tilde{m}_r$, $\bar{b}_t/\tilde{b}_t$, and $\bar{m}_t/\tilde{m}_t$ are given in Table I. It will be seen that these ratios are not constant and that the values quoted by Schneider and by Sidman *et al.* are the values of the ratios at small values of $\bar{b}$. Most cortical sources occur at eccentricities between $\bar{b} = 0.6$ and $\bar{b} = 0.9$; over this range the error in eccentricity introduced by using the constant multipliers may be as much as 8.5 percent of the outside radius of the scalp, as shown in Fig. 2.

The data points illustrated in Fig. 2 turn up to a slope of unity at large values of $\bar{b}$; over this range there is therefore a constant *difference* between $\bar{b}$ and $\tilde{b}$, rather than the constant *ratio* that occurred at low values of $\bar{b}$. This effect is shown both in Table I and in Fig. 3 in which we plot $\bar{b} - \tilde{b}$ versus $\tilde{b}$ for various combinations of skull and scalp thickness. These curves are drawn for a range of skull and scalp thicknesses encompassing the values that are likely to be met in practice [14], [15]. These curves can be used to find the corrected location for any source in a model head, the parameters of which coincide with those used in the graphs. For example, a radial dipole is calculated to be at an eccentricity of 0.60 in a homogeneous head, and we wish to find the correction for a head having a scalp thickness of 0.02 and a skull with parameters thickness = 0.10 and relative conductivity = 0.0125. From Fig. 3, the correction is 0.19, and the corrected eccentricity of the source would be 0.79.

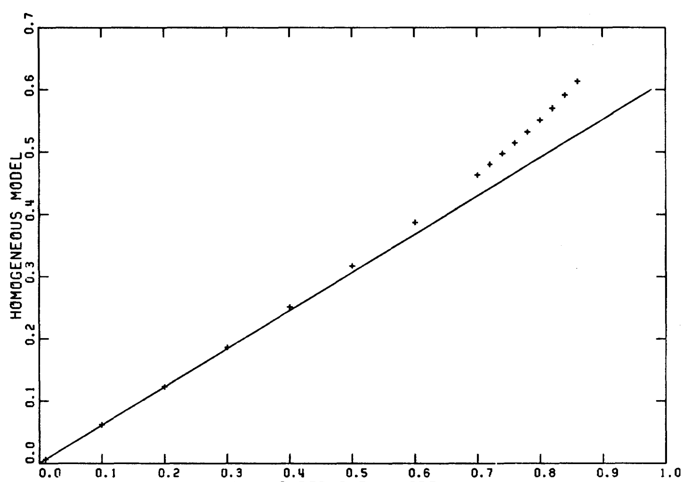

Fig. 2. Eccentricity of a radial dipole source in a homogeneous model head versus the eccentricity of a similar source in a shell model head. The latter eccentricity is calculated to give the least-squared error between the two surface potential distributions. + = model data points. Solid line shows the correction factor proposed by Schneider [9] and by Sidman *et al.* [10].

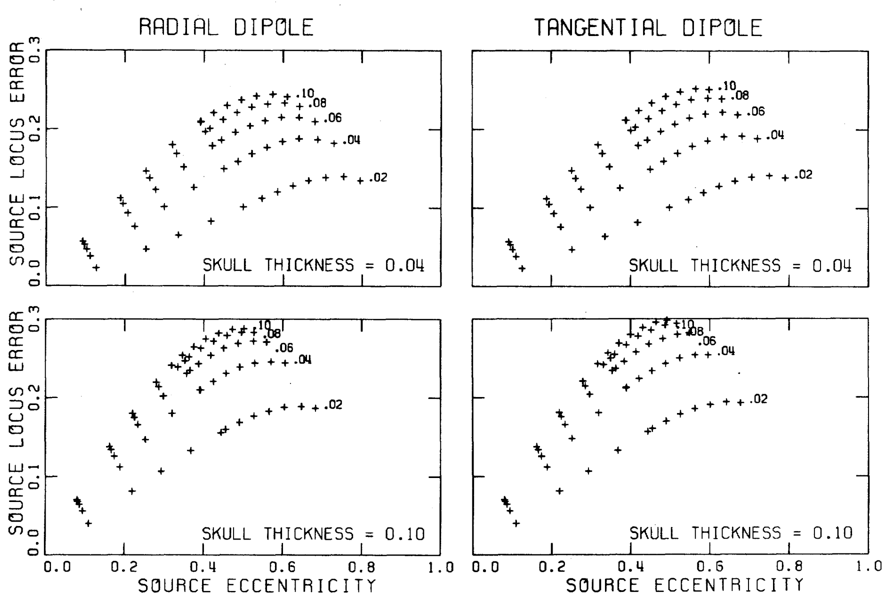

Fig. 3. Horizontal axis: eccentricity of dipole in homogeneous model head. Vertical axis: correction to be added to eccentricity to find location of source in shell model head. The skull thickness is shown on diagrams; conductivity ratio is 0.0125. Scalp thickness is shown to the right of each set of data points. All dimensions given as a fraction of an outside radius of unity.

In work of this sort, the experimenter usually has little idea of the skull and scalp parameters for each subject. Most workers in the past have used the values quoted by Rush and Driscoll [13]. These values lie in the midrange of the skull and scalp parameters used in Fig. 3, and it will be seen from this figure that for cortical sources of eccentricity greater than 0.5, for a given scalp thickness, variations in skull thickness from a middle value within the range quoted by Todd [14] may cause an error of ±7 percent of the outside radius of the scalp in the estimation of the corrected eccentricity; and for a given scalp thickness, variations in skull thickness from a middle value within the range quoted by Todd and Kuenzel [15] may cause an error of ±3 percent. These errors are larger than the best precision that can be achieved in source location calculations; thus, some effort should be made to estimate the skull and scalp parameters for each subject. If the resultant parameters fall between those given in Fig. 3, linear interpolation should be used; this will give a source location correct to ±1 percent of the outside radius of the scalp. The relative conductivity of the skull is very difficult to estimate, but fortunately the source location is not too sensitive to this parameter; evaluation of data similar to those in Fig. 3 but for $\xi = 0.010$ or 0.015 instead of 0.0125 shows that the maximum variation of source location introduced by these extreme values is ±2 percent of the outside radius of the scalp.

These values of ξ refer to the mean value of resistivity averaged over the whole skull, and discount point to point variability; these values span the ranges quoted by Rush and Driscoll [16].

If all skull and scalp parameters are known with some precision, these errors can obviously be avoided by calculating the correction factors from (9), (8), (7), (3a), and (2a), and then finding $\tilde{b}_r$ numerically or from (16), (15), (14), (3a), and (2a), and then finding $\tilde{b}_t$ numerically. This is a tedious calculation but is obviously within the resources of anyone who can already calculate source locations from scalp potential distributions. However, scalp and skull thicknesses are known to vary from location to location over the head, and current techniques for their noninvasive measurement provide only rough estimates. Similarly, the conductivity ratio and the degree of anisotropy in the skull can only be approximated. We estimate that these errors as well as the aspheric shape of the head will reduce the accuracy of the most favorable source localizations to at least ±2 percent of the outside radius of the scalp; thus, more precise correction factors are presently unwarranted.

It will be seen from Table I and Fig. 3 that the corrected parameters for radial and tangential dipoles are slightly different, hence the two components of an oblique dipole map into slightly different corrected locations. The difference in all cases is less than the 2 percent residual error mentioned above, hence an oblique dipole can adequately be handled by linear interpolation between the radial and tangential cases.

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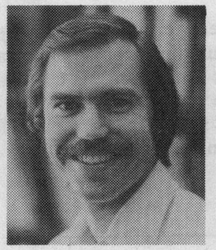

James P. Ary was born in Los Angeles, CA, on April 9, 1947. He received the B.S. degree in physics from St. Mary's College, Moraga, CA, in 1969, studied medical physics at the University of Chicago, Chicago, IL, and received the Ph.D. degree in biophysics from Ohio State University, Columbus, in 1976.

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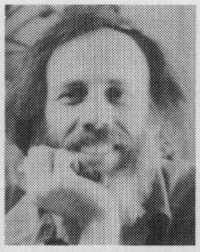

Stanley A. Klein was born in New York, NY, in 1940. He received the B.S. degree in physics from California Institute of Technology, Pasadena, in 1961, and the Ph.D. degree in theoretical particle physics from Brandeis University, Waltham, MA, in 1967.

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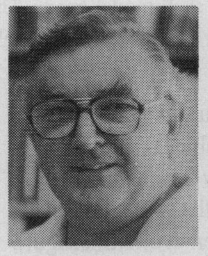

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